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USE OF THRUST MAGNITUDE CONTROL FOR STRATEGIC AND TACTICAL MISSILES

John P. Matuszewski, et al

Aeronautical Systems Division
Wright-Patterson Air Force Base, Ohio

June 1973

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13. ABSTRACT

There has been considerable interest over the past several years in the use of a missile's propulsive thrust not only as the propulsive unit, but also as a basic control mechanism for the vehicle. This paper presents the status of the application of optimal control theory to guidance analysis of advanced air-launched missiles which have the capability to modulate thrust magnitude (TMC). A brief synopsis is given of work which investigates the effect of TMC relative to proportional navigation, optimal turn laws, and overall trajectory optimization. Results presented include typical trajectories, possible control history, work currently in progress, and planned future work. A comprehensive reference list is given.

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FOREWORD

This report was prepared by Captain John P. Matuszewski of the Avionics Subsystems Division, Directorate of Advanced Systems Design, Deputy for Development Planning (ASD/XRHA) and Captain Robert B. Asher of the Air Force Avionics Laboratory (AFAL/NVS-1). The work reported herein was performed under Job Order 611A0G202000A, Strategic Bomber Penetration Analyses. The period covered by this report is January 1971 through April 1973. This report was submitted by the authors on 9 April 1973.

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This technical report has been reviewed and is approved.

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Deputy for Development Planning

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There has been considerable interest over the past several years in the use of a missile's propulsive thrust not only as the propulsive unit, but also as a basic control mechanism for the vehicle. This paper presents the status of the application of optimal control theory to guidance analysis of advanced air-launched missiles which have the capability to modulate thrust magnitude (TMC). A brief synopsis is given of work which investigates the effect of TMC relative to proportional navigation, optimal turn laws, and overall trajectory optimization. Results presented include typical trajectories, possible control histories, work currently in progress, and planned future work. A comprehensive reference list is given.

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SECTION I

INTRODUCTION

There has been considerable interest over the past several years in the use of a missile's propulsive thrust, not only as the propulsive unit, but also as a basic control mechanism for the vehicle. This philosophy is being considered for use in several strategic and tactical missile programs within the Air Force and Navy. Examples include the Navy Agile Missile Program and the Air Force Bomber Defense Missile and Multi-Mission Missile Programs.

One method by which the thrust is used as a control mechanism is by using the force available to rotate the missile. Viable methods for accomplishing this thrust vector control (TVC) include thrust jet deflection tabs, sideforce control jets, and gimballed nozzles.

The thrust level may be zero, i.e., the motor turned off; may be at its maximum thrust level, T_{max}; or may be burning at any intermediate thrust level, T, between the two extreme limits, $0 < T < T_{max}$. The modulation of the thrust level between its two limits is referred to as thrust magnitude (or modulation) control (TMC). The use of an intermediate thrust level establishes the segment of the trajectory referred to as an intermediate thrusting arc.

One of the basic questions asked in the use of thrust magnitude control is whether and how TMC will improve missile performance for a variety of missions. One approach to answering some of the questions is by using optimal control theory to determine thrust control. Many papers have appeared in the literature (e.g., 7, 8, 9, 13, 19) examining optimal thrust control for aircraft and rockets. The papers verify the fact that there is a strong probability of singular arcs (20, 24, 25); i.e., an intermediate thrusting arc will be optimal.

Even though the above applications papers are important in their own right, simplifying assumptions have been made. Modifications in the control laws are expected when more realistic analysis is performed. More realistic analysis is necessary for state-of-the-art missile applications for several reasons. The miss distance requirements for several missiles are becoming more stringent. These requirements dictate that the terminal guidance law mechanization be accurately implemented. However, unless variable thrusting was available to obtain the required axial acceleration the missile slowdown caused by terminal maneuvers would increase the miss distance. Intermediate thrusting may increase the missile maneuverability by being able to command a normal acceleration higher than

that obtainable by aerodynamic forces. This would allow one to increase the high altitude performance of the missile where aerodynamic forces are not as effective. Thus, the missile effective time constant can be decreased. As mentioned previously, the possibility of singular arcs in the minimum time and maximum range problems exists. Thus, the optimality of intermediate thrusting may allow the intercept problem to better accomplish these two objectives. Another problem inherent in high Mach number missiles with radar guidance systems (whether active or semiactive) is that the radome cannot withstand the integrated thermal heat input and cannot withstand a thermal shock induced by a rapid heat input. The thermal shock induces radome stresses large enough to crack and destroy the radome. The increased temperature may exceed the thermal limits of the radome causing destruction. Modulating the thrust may allow a better behaved trajectory that will allow the radome to withstand the thermal environment.

There are inherent penalties with the use of today's technology in thrust magnitude control, however. The cost of the basic rocket motor is higher than that of more conventional systems. The mechanization requires larger sizes of motors per unit of specific impulse. However, the performance gains in a particular application may dictate the use of TMC.

This paper presents the results of contract and in-house work relative to the control problem for missiles which have the TMC capability, describes the current effort being conducted by the authors, and indicates future work planned in the area.

SECTION II

PROBLEM FORMULATIONS AND RESULTS

There have been several Air Force contracted and in-house efforts designed to study optimal intercept for air-to-air missiles with thrust magnitude control as a primary control variable. The problem formulations of this work have taken various forms depending upon several factors: different launch conditions; objectives functions; and constraints. The formulations are discussed here along with the results obtained.

1. THRUST MAGNITUDE CONTROL AND PROPORTIONAL GUIDANCE

In order to maintain a proportional guidance law for target intercept in the air-to-air missile problem one should establish a longitudinal acceleration. Reference (4) studies the use of thrust magnitude control to obtain this acceleration component. This allows the missile's corrective acceleration to be maintained perpendicular to the line-of-sight.

Ideally, the commanded acceleration used to null the inertial line-of-sight (LOS) rate, is to be perpendicular to the LOS vector. When body normal aerodynamic forces are used alone, a component which varies as the cosine of the gimbal angle provides the desired corrective acceleration. The other component, which varies as the sine of the gimbal angle, causes an undesired variation in closing speed.

One may consider the component of acceleration normal to the line-of-sight (a_n) and the component of acceleration along the line-of-sight (a_r). In order to implement proportional guidance, it is necessary to command the missile acceleration components as

$$a_n = k\, v_c\, \dot{\sigma}$$

$$a_r = 0$$

where k is the navigation gain, v_c is the closing velocity, and $\dot{\sigma}$ is the line-of-sight rate. The equations transforming accelerations along the control axes to the desired accelerations along the guidance command coordinate system (see Figure 1) can be obtained as

$$a_n = \left(\frac{F_x}{m}\right) \sin \xi - \left(\frac{F_y}{m}\right) \cos \xi = k\, v_c\, \dot{\sigma}$$

$$a_r = \left(\frac{F_x}{m}\right) \cos \xi + \left(\frac{F_y}{m}\right) \sin \xi = 0$$

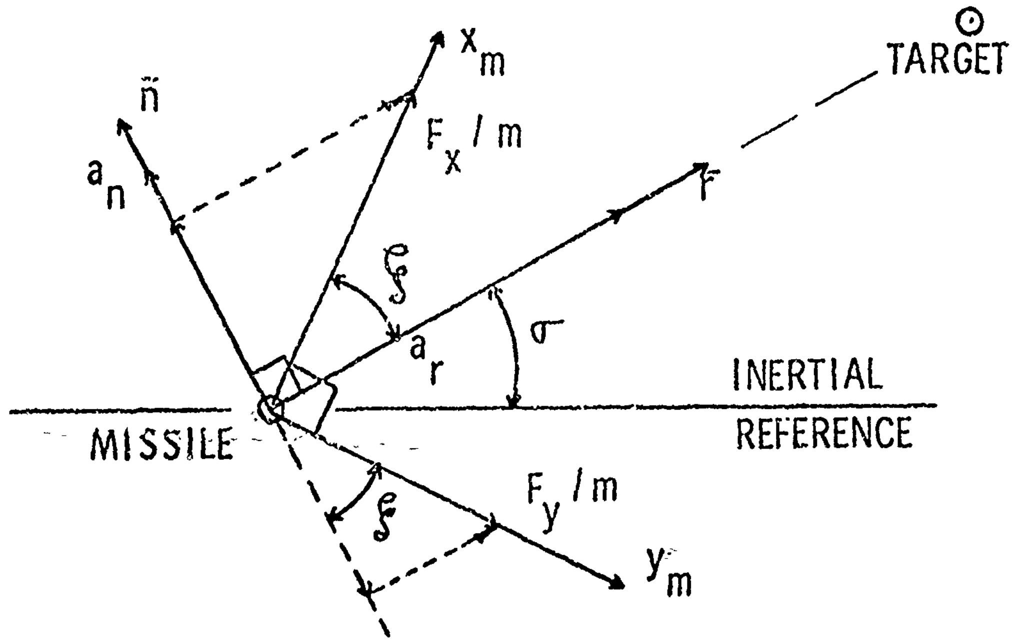
One may solve for $\left(\frac{F_x}{m}\right)$ and $\left(\frac{F_y}{m}\right)$ and obtain:

$$\frac{F_x}{m} = (k\, v_c\, \dot{\sigma}) \sin \xi = a_n \sin \xi$$

$$\frac{F_y}{m} = -(k\, v_c\, \dot{\sigma}) \cos \xi = -a_n \cos \xi$$

Thus, if the missile's longitudinal and normal accelerations are obtained as the above, all missile accelerations will be perpendicular to the line-of-sight and the component of closing speed contributed by the missile will be kept constant.

FIGURE 1: AXIS SYSTEM ORIENTATION



The net payoff for TMC for the problem is as follows: (a) TMC allows the proportional guidance law to be followed exactly; (b) TMC minimizes the variation in the closing speed of the engagement; (c) TMC increases the intercept zone by providing the needed corrective accelerations at large gimbal angles; (d) TMC reduces the guidance system time constant by providing an increase in turn rate without an increase in angle-of-attack.

The study referenced shows how TMC can be used to obtain a more accurate implementation of the proportional guidance law and, thus, obtain an optimal intercept. However, one must carefully consider the optimality of proportional guidance as the optimality is based on several assumptions (26, 11, 12).

2. TURN LAW OPTIMIZATION

The turn law optimization work concentrates on obtaining control laws for the missile immediately after launch. After the missile has achieved the desired heading and velocity some other guidance law will be used. The control problem for the turn consists of finding the control law to turn the missile to a desired attitude up to 180° from launch altitude.

The first reference in this category (3) solved the trajectory optimization problem for an idealized missile in horizontal planar flight by analytic application of the maximum principle. Simplified missile equations of motion were used:

$$\dot{V} = \frac{1}{m} (T \cos \alpha - D)$$

$$\dot{\gamma} = \frac{1}{mV} (T \sin \alpha + L)$$

$$\dot{m} = -f|T|$$

where:

V = missile speed

γ = missile flight path angle

m = missile mass

T = missile thrust

α = missile angle-of-attack

D = missile drag

L = missile lift

f = fuel flow constant

t_f = final time

It is assumed that some fixed final value of γ is to be reached ($\gamma(t_f) = \gamma_f$) at the end of turn and that time, speed, and mass may or may not be constrained.

The cases considered are tabulated in Table 1. The results obtained from this study include the following: (a) in every case thrust is either full on or full off, throttling is never of value; (b) when power is off, the angle-of-attack is such as to maximize the ratio of lift-to-drag except for cases 3 and 7. In case 7 the angle-of-attack is arbitrary, i.e., one simply turns to the desired heading with engine off, in any manner. In case 3 the vehicle's mass enters into the calculations of coast angle-of-attack; (c) in all cases where there is a coast, the coast precedes the burn.

If the problem of case 2 is considered in more depth, it is found that for minimum time turns the thrust should be on continuously and that a constant attitude turn law closely approximates the time-optimal flight-path-angle trajectory. Typical optimal normalized trajectories are shown in Figure 2.

TABLE 1: TURN LAW OPTIMIZATION CASES

CASE	CRITERION	FINAL SPEED CONSTRAINED	FINAL MASS CONSTRAINED
1	MINIMUM	NO	NO
2	TIME	YES	NO
3		YES	YES
4	MAXIMUM	NO	YES
5	FINAL	NO	NO
	VELOCITY		
6	MAXIMUM	YES	NO
7	FINAL	NO	NO
	MASS		
8	MAXIMUM	NO	NO
	BURNOUT		
	VELOCITY		

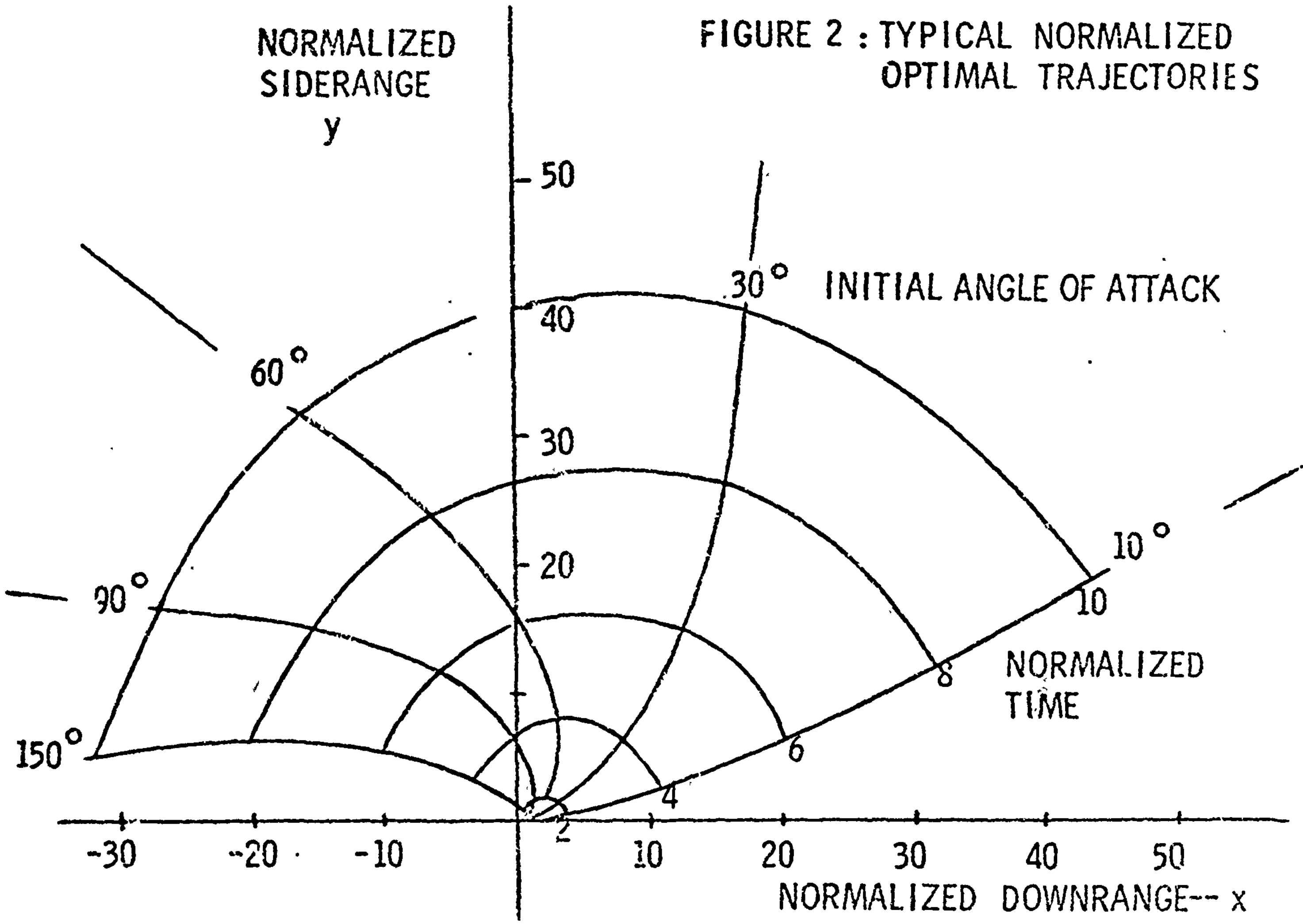


FIGURE 2 : TYPICAL NORMALIZED OPTIMAL TRAJECTORIES

The second work in the turn-law category (1) considered other predetermined turn laws in addition to constant attitude, and compared them to time-optimal and minimum-impulse turns. It was found that depending on final velocity required at the end of the turn, a constant angle-of-attack turn law as well as a constant body attitude turn law with maximum thrust closely approximated the time-optimal turn. The magnitude of the constant angle-of-attack will be different for different ranges of launch velocity.

If a minimum-impulse turn is desired, the thrust profile for the turn is shown to be a coast-boost during the turn as in Figure 3.

There is, in fact, a compromise between time-to-turn and propellant weight expended for various thrust control policies. This tradeoff is depicted for three thrust policies in Figure 4. As is seen, the all-boost motor with a minimum-time-to-turn angle-of-attack provides the shortest turn time. However, a coast-boost thrust profile yet minimum time angle-of-attack during the turn would allow less fuel consumption during the turn and take longer to complete the turn.

An interesting aspect of the above results is that no singular arcs were observed. Indeed, in (1) it is shown that the strengthened Legendre-Clebsch condition is, in fact, satisfied for the aerodynamics and velocity ranges considered. Thus, no singular arcs should occur for the turn.

Other analyses in the literature indicate strong possibilities for the existence of singular arcs when optimization over the entire trajectory is considered rather than just the turn. This is considered in the next section.

3. OPTIMIZATION FOR OVERALL TRAJECTORY

The Green's theorem approach is used in (5), but determination of optimal control is done only for horizontal rectilinear flight. The problem considered was that of maximizing range in a given time (or equivalently minimum time for a given range). The result is that a typical optimal thrust profile consists of a boost period of full thrust, followed by a sustain period of approximately constant thrust until all fuel is consumed, followed finally by a coast period of zero thrust. For the preliminary case considered in this work, the sustain period corresponds to a singular arc.

Both sideforce control and thrust control are chosen in (6) for maximization of launch range for intercept of non-maneuvering targets.

FIGURE 3: COAST-BOOST DURING TURN

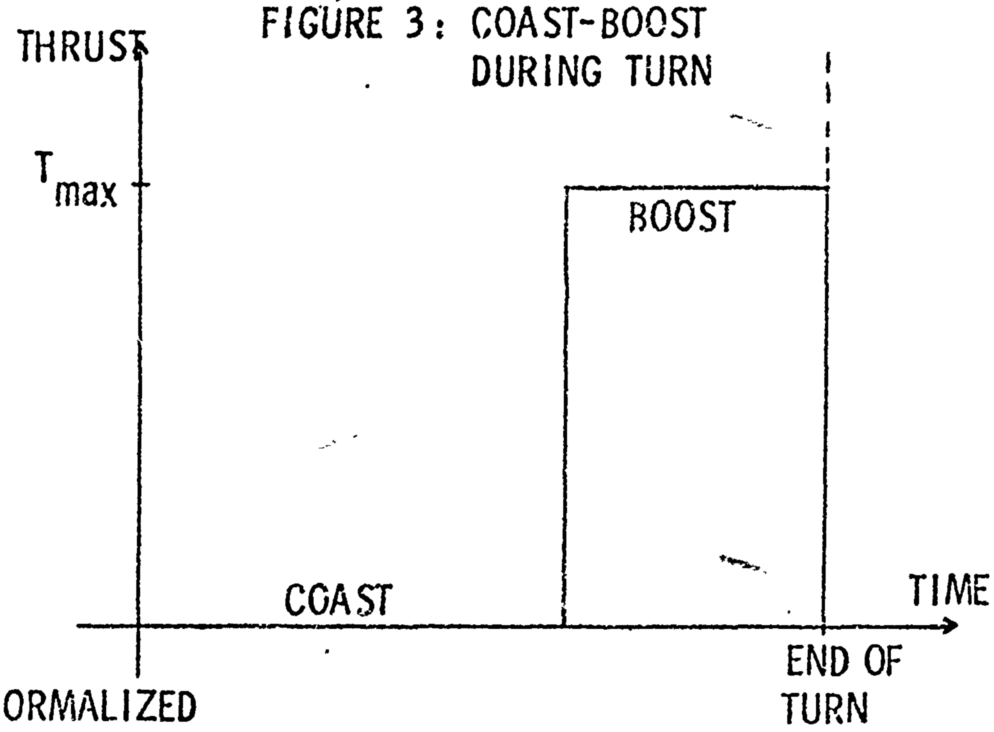
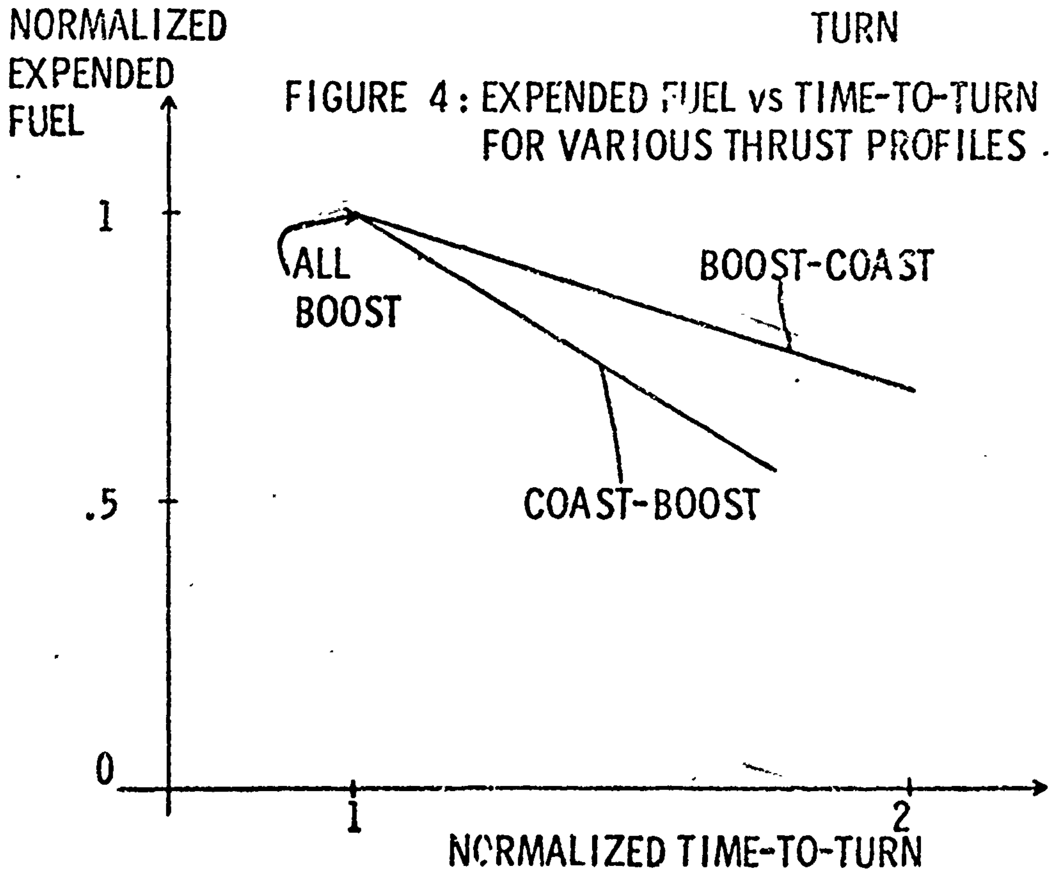


FIGURE 4: EXPENDED FUEL vs TIME-TO-TURN FOR VARIOUS THRUST PROFILES.



In this work it is shown that the function of lateral control (sideforce) is to get the missile onto a straight-line collision course with the target as soon as possible after launch. Thereafter the sideforce control is zero (for non-maneuvering target) and a singular fuel burning control is observed. The missile velocity is constant on these singular arcs and depends only on the effective exhaust velocity of the missile and the component of target velocity in the direction of the final missile velocity vector. The fuel burning rate, however, can be nonconstant on the singular arc if the drag coefficient changes along the trajectory.

The work of (15, 16) adds the complication of a maneuvering target into the problem formulation of (6). Results are presented utilizing an adaptive feedback scheme for the optimization. Intercept was achieved against a target executing a maneuver unknown a-priori to the pursuing missile. The results of this simulation substantiates the results of (6) for sideforce control and includes more. That is, in all cases in (6) it was desirable to null out initial errors by large initial sideforce in an effort to put the pursuing missile on a straight line collision course with the target then reduce sideforce to zero. In the present work, since the target is maneuvering, the sideforce did not reduce to zero. The sideforce control obtained by the suboptimal feedback scheme for the case is shown in Figure 5; i.e., the non-zero sideforce is required to counteract the target maneuver.

The result for thrust control, however, is substantially the same as in previous work in that, in general, a boost-sustain type of profile is to be used until burnout. One distinction made in these latest works is that although minimum time or minimum impulse provide rational choices for performance indices, the other constraints on the intercept problem dictate motivation for thrust modulation, not necessarily performance improvement. One of these constraints, which will be discussed again later, is the existence of a velocity limit due to radome heating. The result in (16) shows a feedback scheme which can be used to strive for constant velocity intercepts even with maneuvering targets.

A last remark to be made relative to (15, 16, 17) is that if the intercept is of such a nature that some fuel can be saved, the accepted thrust profile of Figure 6 (a) could be modified to that of Figure 6 (b), 6 (c), 6 (d); i.e., a boost-sustain-boost or a multi-pulse. The motivation for policy of Figure 6 (b), 6 (c) is that near the end of intercept the seeker limitations make active use of guidance impractical. Thus, the objective of policy 6 (b) or 6 (c) would be to increase missile velocity at the last measured heading to shorten the time the target has to maneuver away. Policy 6 (d) could be used for velocity limiting.

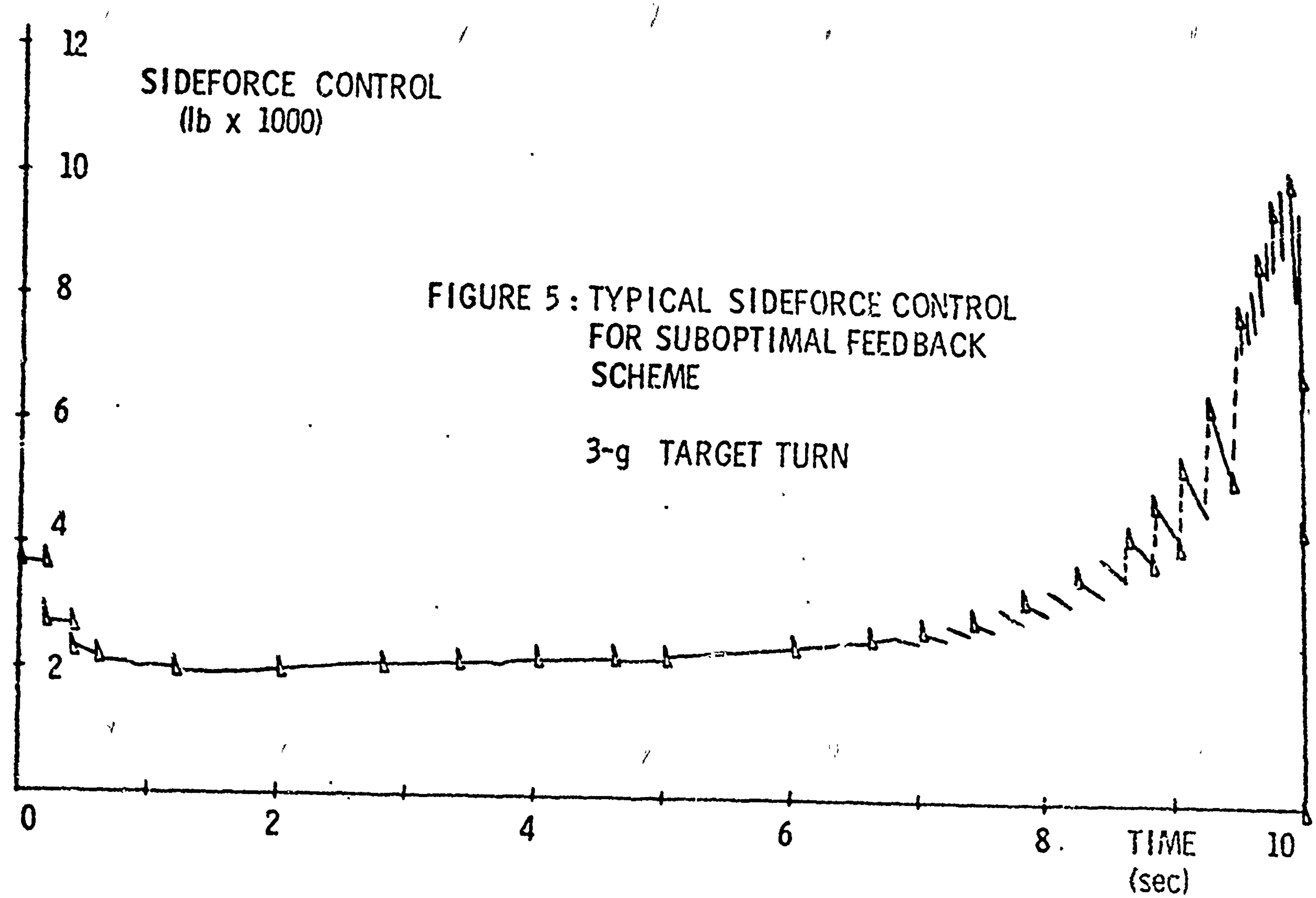
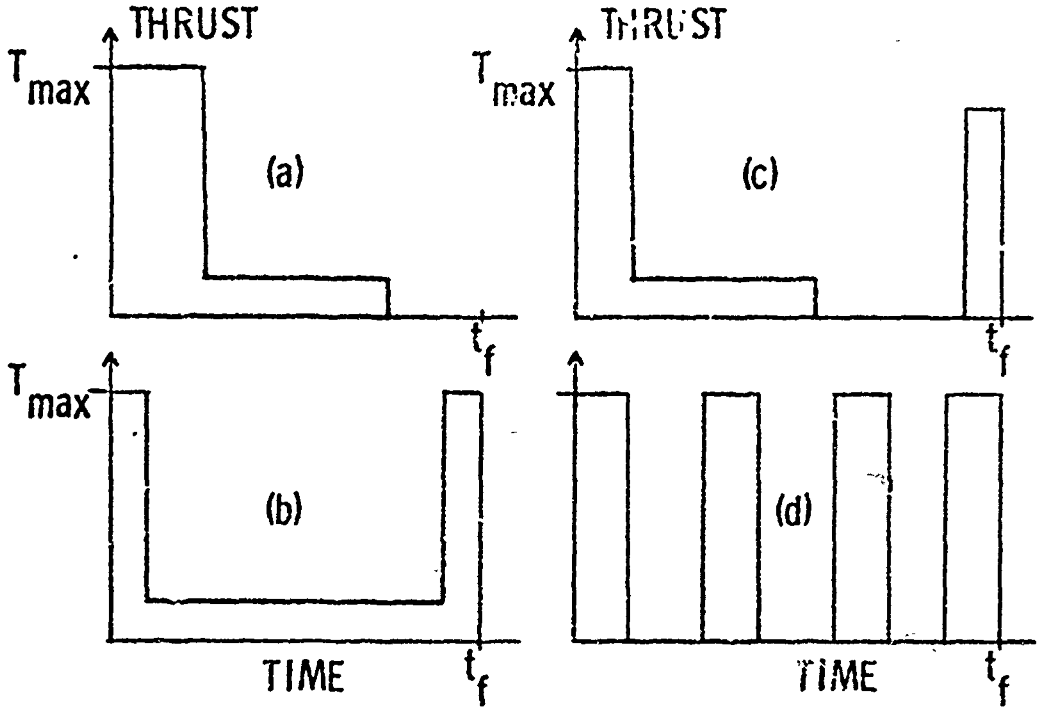


FIGURE 5: TYPICAL SIDEFORCE CONTROL FOR SUBOPTIMAL FEEDBACK SCHEME

3-g TARGET TURN

FIGURE 6 : ALTERNATIVE THRUST PROFILES



Alternatively, based on the 3-d proportional navigation analysis (4) having thrust control for the last few instants of the intercept can significantly reduce miss distance.

It is seen then that there have been many theoretical investigations regarding how best to utilize TMC for intercept missiles. What is actually used for any missile, if any TMC should be used at all, will more likely be governed by what can actually and economically be built in addition to the mission to be accomplished. These above studies, however, serve to indicate what can be done and possible goals that can be sought.

In addition to these last remarks, a further criticism of the existing work is the fact that to get at least preliminary answers many simplifying assumptions have been introduced by the various authors. The last study to be discussed (18) is being performed in order to obtain results without the simplifying assumptions and by incorporating other factors and constraints which make the problem more realistic.

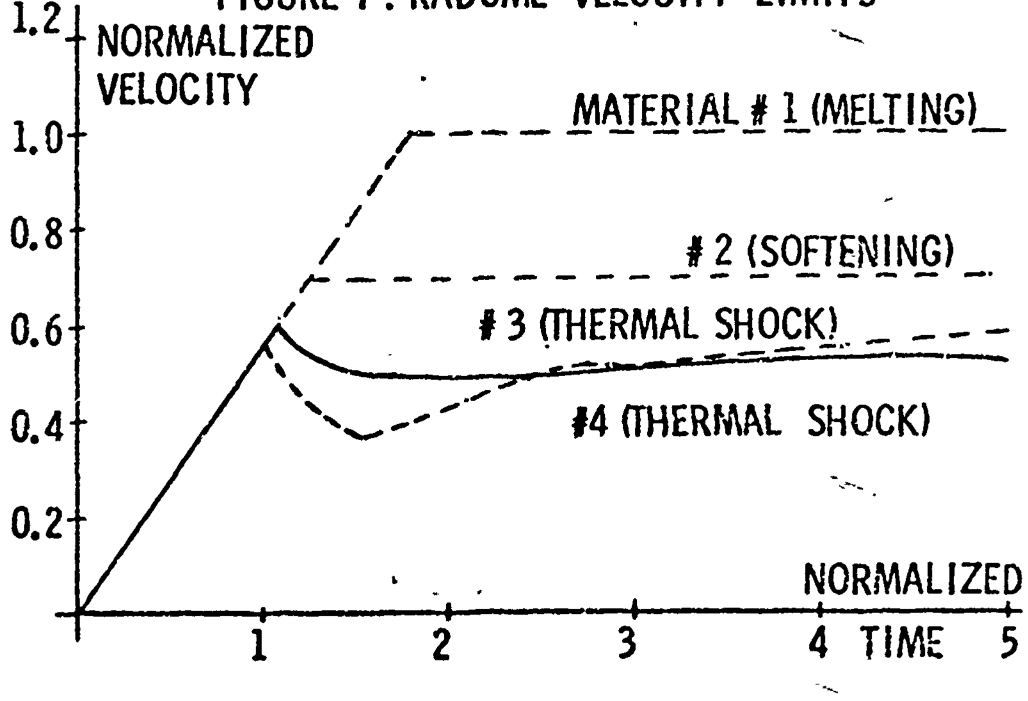
4. COMPLETE OPTIMIZATION

The problem being considered in this work (18) will initially use each of three performance indices: minimum time, maximum range, and their combination. A six degree of freedom model is being used including the latest aerodynamic data for a candidate air slew missile.

One result of the study is the determination of how best to utilize the combination of thrust magnitude, thrust vector, as well as aerodynamic control variables.

To indicate the types of constraints, one which has not been incorporated previously, is a limit on slew rate due to gyro rate limitations. Another limitation is placed on roll rates. A third constraint considered in this work that has not been directly included previously in any analytic optimization work, is a constraint on radome heating and thermal shock. Curves showing static velocity limits as a result of melting and shock limitations are shown for typical materials in Figure 7.

FIGURE 7: RADOME VELOCITY LIMITS



As a first approximation a velocity limit can be established for the optimization problem. But a model for radome heating and thermal shock is more accurate and has been incorporated in this study. Lastly, in addition to the complication that all these constraints are included simultaneously, one is faced with control and state-variable equality and inequality constraints and, of course, the probability of singular arcs. The adjoint equations are modified as a result of the equality constraint on a subset of the control variables; the Jacobson method (20) is used in conjunction with an optimal steepest descent algorithm (21) in order to compute solutions which may include singular arcs.

The status of the work as of this writing is that the problem has been completely formulated and is being programmed.

SECTION III

FUTURE WORK

The current work being conducted by the authors (18) will be exercised initially for the Bomber Defense Missile and possibly for other high performance missiles. The purpose is to evaluate the performance for various missions when using several control variables in combination, including TVC and TMC.

Further work will be accomplished in developing implementable control laws using the results of the current TMC study. This will be accomplished by using and developing required approximation techniques for obtaining feedback guidance schemes.

SECTION IV

SUMMARY AND CONCLUSIONS

This paper presents a review of the state-of-the-art in application of optimal control for missiles equipped with a thrust magnitude control (TMC) capability.

It has been shown by the authors of the papers discussed and the present authors that TMC is a viable candidate for state-of-the-art missile applications. The use of TMC may be justified not only by intercept performance improvement such as minimum time, but also by the capability of TMC to compensate for basic missile configuration constraints.

The advantages of TMC include: (1) A reduced miss distance has been shown for the case when proportional navigation missile guidance is used and is expected for other guidance laws; (2) Increased range; (3) Added missile maneuverability for thrust vector controlled missiles by having some thrusting force available for longer durations than current missiles; and (4) Velocity control to counteract such limiting configuration constraints as radome heating and thermal shock.

While many questions regarding the use of TMC for advanced air-launched missiles have been answered by the work discussed in this paper, there are many more which arise when an application is considered in depth. Work is continuing in the area in an effort to cope with these problems as they arise.

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